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ed,the $ has *not*. The $ has not entered into the division at all. It is simply annexed to the 6. The principle is evident.

ARE DIFFERENTIALS FINITE QUANTITIES?

By JOHN N. LYLE, Ph. D., Professor of Mathematics, Westminster College, Fulton, Missour..

In seeking for a correct answer to the above question let us reconsider in detail a familiar elementary example.

Let $u=x^2\ldots(1)$, in which u is a function of the independent variable x that increases in value *uniformly*.

The *increment* of the variable x in *a unit of time* is the *rate of variation* of the variable x and may be appropriately represented by the symbol dx.

When the variable x reaches the value x' or AB, Fig. 1, and the function u, the corresponding value u' or AC we shall have $u'=x'^2\ldots.(2)$.

Let $\Delta x=\dfrac{dx}{n}$, in which Δx represents the increment of the variable x in $\dfrac{1}{n}$ of a unit of time. Since x varies uniformly, $n\times\Delta x$ will equal dx; that is, will equal the rate of variation of the independent variable x.

When the variable x reaches the value x''; that is, $x'+\dfrac{dx}{n}$, or Aa, and the function u, the corresponding value u'' or Ai we shall have $u''=x''^2=\left(x'+\dfrac{dx}{n}\right)^2\ldots.(3)$.

Subtract (2) from (3).

Then $u''-u'$, or $\Delta u'=x''^2-x'^2=2x'\dfrac{dx}{n}+\dfrac{dx^2}{n^2}\ldots.(4)$.

Multiply both members by n. Then $n\times\Delta u'=2x'dx+\dfrac{dx^2}{n}\ldots.(5)$. The second member of (5) is made up of two parts, one of which, $2x'dx$, is constant whatever the value of n may be, and the other, $\dfrac{dx^2}{n}$, decreases without limit as n increases without limit. $\dfrac{dx^2}{n}$ is n times $\dfrac{dx^2}{n^2}$, the addition to the increment of the function in $\dfrac{1}{n}$ of the unit of time due to the *increase* in the tendency of the function to vary after passing the value u'.

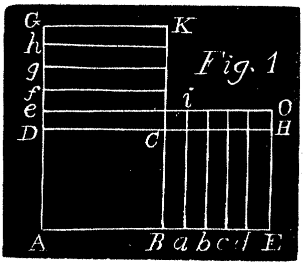

Fig. 1

$\dfrac{2x'dx}{n}\times n$, that is, $2x'dx$ is the increment of the function u in the unit of time that is due to its tendency to vary when it reaches the value u'.

The increment $2x'dx$ is received *uniformly* during the unit of time and symbolizes the rate of variation of the function u when it reaches the value u'.

Let $i=\frac{dx^2}{n}$....(6). Subtract (6) from (5).

Then $n\times\Delta u'-i=2x'dx$.....(7). Since $2x'dx$ symbolizes the rate of variation of the function u corresponding to the value u',

$n\times\Delta u'-i$ must symbolize the same thing.

Instead of $n\times\Delta u'-i$ write du' and we shall have $du'=2x'dx$...(8), in which du' represents the rate of variation of the function u corresponding to the value u'. du' is, also, seen to be the limit that the variable product $n\times\Delta u'$ approaches as n increases without limit and $\Delta u'$ decreases without limit.

The geometrical value of dx is the line BE or DG in Fig. 1. But each of these lines has two ends and is, therefore, *finite*. The geometrical value of x' is the line BC or DC.

As each of these lines has *two ends* they are *finite*.

It follows that $2x'dx$, that is, du' is *finite*. The geometrical expression for $2x'dx$ is the sum of the two rectangles CE and CG.

By dropping the primes in (8) we have the general expression $du=2xdx$....(9), in which dx stands for the differential or rate of variation of the function u.

By hypothesis dx is *a constant*. Furthermore, the values x',x'',x''',&c., attributed to the variable x are *finite*. Hence, the values of du corresponding to the different values of the variable x are *finite*.

Remark 1. $\Delta u'$ is the increment of the function u in $\frac{1}{n}$ of the unit of time after reaching the value u'.

The geometrical value of $\Delta u'$ is the sum of the two rectangles Ca and Ce and the square Ci.

$n\times Ca=CE$, $n\times Ce=CG$, and $n\times Ci=CO$.

Equation (5) is obtained by multiplying the increment of the function u in $\frac{1}{n}$ of the unit of time by n. This is the data from which we deduce the inference that the rate of variation of the function u corresponding to the value u' is $2x'dx$, the limit of $n\times\Delta u'$ as n increases without limit and du diminishes without limit.

Since it would be absurd to destroy the premises of an argument in order to reach a logical conclusion therefrom, so it would be illogical and absurd to annihilate $\Delta u'$ in order to obtain du'.

Remark 2. A *finite* straight line is one that has a beginning and a termination, that is, two ends.

A *finite* series is one that has a first and a last term.

A *finite* number is one whose units would constitute a series having a first and a last term.

Remark 3. Since the geometrical value of dx is the finite line BE, $\frac{1}{n}$

of *dx*, or Δx, is a part of *BE* having two ends and is, therefore, *finite*.

If the successive values of n are the numbers 1, 2, 3, 4, 5, &c., of the counting series, they are finite numbers inasmuch as the units of each of these numbers would constitute a series having a first and a last term.

Remark 4. Ci is the geometrical equivalent of $\frac{dx^2}{n^2}$, or Δx^2; and $n \times Ci$ or CO is the geometrical equvivalent of $\frac{dx^2}{n}$, or $n\times\Delta x^2$.

As n increases without limit Ci and CO diminish without limit.

The rectangles Ci and CO have finite areas inasmuch as their sides are finite lines. Diminishing their value neither annihilates them nor changes their character as finite quantities.

Remark 5. Let $x=12$ inches, $dx=1$ inch and $n=10$.

Then will $\Delta u'=24\times.1+.1^2=2.41$, and $n\times\Delta u'=10\times2.41=24.1$.

If $n=100$: We have $\Delta u'=24\times.01+.01^2=.2401$, and $n\times\Delta u'=100\times.2401=24.01$.

If $n=1000$: Then $\Delta u'=24\times.001+.001^2=.024001$, and $n\times\Delta u'=1000\times.024001=24.001$.

The variable product $n\times\Delta u'$ whose successive values are 24.1, 24.01, 24.001, &c., approaches the limit 24 which is the rate of increase of the area when the side is 12 inches and its rate 1 inch per second. du' is the limit of $n\times\Delta u'$ and its value is 24.

The numerical values attributed to n and Δx are, likewise, *finite*.

Remark 6. Equation (7) is derived from equation (5) in accordance with Euclid's axiom, 3,—If equals be taken from equals, the remainders are equal.

Berkeley, Carnot, and Compte refer to a compensation of errors in the Calculus.

This compensation is really accounted for by Euclid's axiom 3, since i the difference between $n\times\Delta u'$ and du' is equal to $\frac{dx^2}{n}$ the difference between the second member of (5) and $2x'dx$.

* * *

NOTE ON THE CENTROID OF PLANE AREAS.

By Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Let it be required to find the centroid of the area of the curve $r^{\frac{1}{m}}=a^{\frac{1}{m}}\cos\frac{\theta}{m}$, when the density varies as the nth power of the distance of any point from the origin.